DIFFERENTIAL EQUATIONS

An equation that contains derivatives is k/a Differential Equation.

e.g. $\boxed{\frac{dy}{dx} + 2y = \cos x}$

* **Order of the differential Equation:-** It is the order of the highest derivative term present in the differential equation.

* **Degree of the differential equation:-** Degree is the power of the highest order derivative term present in the differential equation.

$$\boxed{\frac{dy}{dx} + 2y = \cos x}$$

$\Downarrow$

1^{st} order, 1^{st} degree differential equation.

(1) $3y = x\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right)$

order = 1, Degree can not be determined.
because it is an infinite series.

$$3y = x\frac{dy}{dx} + \left[\frac{dy}{dx} - \left(\frac{dy}{dx}\right)^3 \frac{1}{3!} + \cdots\right]$$

(2) $\sqrt{2\left(\frac{dy}{dx}\right)^3 + 4} = \left(\frac{d^2y}{dx^2}\right)^{3/2}$

order = 2, Degree = ~~$\frac{d^2y}{dx^2}$~~ 3, differential equation.

* First Order Differential Equation:-

Any first order differential equation can be written as-

$$\frac{dy}{dx} = f(x,y)$$

$$\Downarrow$$

$$\Rightarrow y = f(x) \longleftarrow \text{Solution.}$$

* Physical Interpretation:-

If $y = f(x) \longrightarrow$ curve

$\frac{dy}{dx} \Rightarrow$ slope of tangent to the curve.

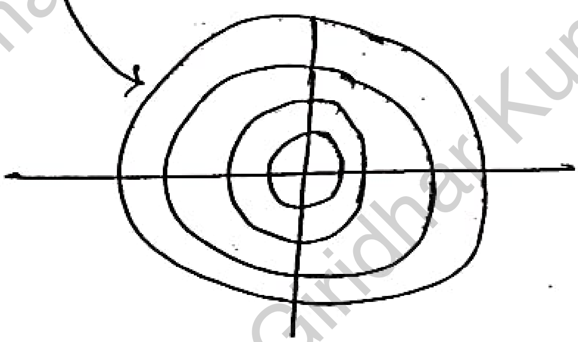
Here $\boxed{y = f(x)} \longrightarrow$ It denotes the family of curve.

$\boxed{\frac{dy}{dx} = f(x,y)} \longrightarrow$ slope of tangent to the curve.

Let an equation $x^2+y^2=c$, this equation represent family of circle of centere at $(0,0)$ and radius is $\sqrt{c}$. It is not a single circle, it is family of circle.

$\boxed{x^2+y^2 = c}$

$(0,0)$, $r = \sqrt{c}$



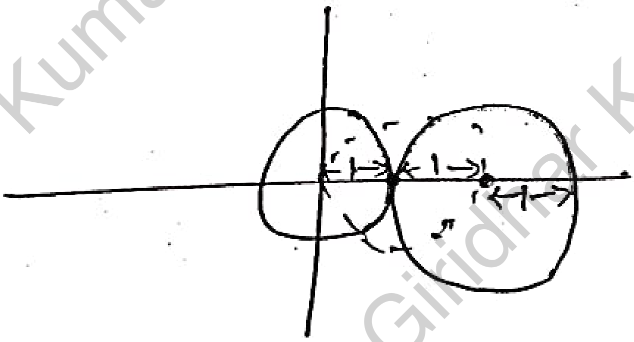
Let an another equation -

$$(x-h)^2+(y-k)^2=a^2 \quad \text{here } r=a=\text{Constt.}$$

It represents family of circles of curve of radius 'a' and centre at (h,k) which is variable. So centre may change.

When

$$(x-h)^2+(y-k)^2=1$$



Case - I Separation of Variables :-

$$\frac{dy}{dx}=f(x,y)=\frac{\phi_1(x)}{\phi_2(y)}$$

$$\Rightarrow \boxed{\int \phi_1(x)\,dx=\int \phi_2(y)\,dy}$$

A-7

Q.1 (ii) Solve the Differential equation (Separation of variables):-

(ii) $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$ given that $y\left(\frac{\pi}{3}\right)=0$

$$\Rightarrow \int \frac{\cos x}{\sin x}dx+\int \frac{e^y}{\underbrace{(e^y+1)}_{=q}}dy=0$$

let $\sin x=p$

$\cos x\,dx=dp$

and $e^y+1=q$

$e^y dy=dq$

$$\Rightarrow \ln(\sin x)+\ln(e^y+1)=\log c$$

$$\Rightarrow \ln\left[\sin x(e^y+1)\right]=\ln c$$

$$\Rightarrow \boxed{\sin x(e^y+1)=c}$$

$\because\ x = \frac{\pi}{3},\ y = 0$

$\therefore\ C = \sin\frac{\pi}{3}(e^0+1)$

$C = \sqrt{3}$

$\Downarrow$

So $\boxed{\sin x(e^y+1) = \sqrt{3}}$ Ans

(iv) $\frac{dy}{dx} = (4x+y+1)^2$ given that $y(0) = 1$

let $4x+y+1 = z$

$4 + \frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dy}{dx} = \frac{dz}{dx} - 4$

Now

$\frac{dz}{dx} - 4 = z^2$

$\frac{dz}{dx} = 4+z^2$

$\int\frac{dz}{4+z^2} = \int dx$

$\int\frac{dz}{2^2+z^2} = \int dx$

$\frac{1}{2}\tan^{-1}\frac{z}{2} = x + C$

$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x+C$

when $x=0,\ y=1$

$\frac{1}{2}\tan^{-1}\frac{2}{2} = 0+C$

$\frac{1}{2}\tan^{-1}\tan\frac{\pi}{4} = C$

$\therefore\ \boxed{C = \frac{\pi}{8}}$

So $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x+\frac{\pi}{8}$

$\Rightarrow \boxed{\tan^{-1}\left(\frac{4x+y+1}{2}\right) = 2x+\frac{\pi}{4}}$

A-6

Q.63 The solution of the D.E. $\frac{dx}{dt} = x^2$ with the initial condition $x(0) = 1$ will blow up as t tend to -

(a) 1 (b) 2 (c) 1/2 (d) ∞.

Sol^n $\frac{dx}{dt} = x^2 \quad x(0) = 1$

$\int\frac{dx}{x^2} = \int dt$

$\Rightarrow -\frac{1}{x} = t+c$

$x(0) = 1 \Rightarrow -1 = 0+c \Rightarrow c = -1$

$-\frac{1}{x} = t-1$

$\Rightarrow \frac{1}{x} = 1-t$

$\Rightarrow \boxed{x = \frac{1}{1-t}}$

So at $t=1$ x blow up (means tends to ∞).

So $\boxed{t=1}$ ans